A Model for Multiphase Flow in Porous Media

The Relevance of Capillary Pressure for the Concept of Relative Permeability

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Abstract

The article describes the correlation between capillary pressure and relative permeability in porous rocks. The Darcy equation for single phase flow and the physics of capillary pressure are shortly described using the concept of the hydraulic radius. Separating two phase flow in a porous medium into a Darcy component and into a capillary pressure component leads to an equation for the relative permabilities.

Examples are given for relative permeabilities in surfactant flooding, the change in wettability and dependence of relative permeabilities on pressure or flow velocity.

Introduction

The description of the flow of liquids and gases in porous rocks is the basis of all calculations in reservoir engineering. The flow of fluids in porous rocks is described in the petroleum industry by empirically derived laws and equations, as the Darcy equation for the general flow description and the concept of relative permeabilities for multi phase flow. All these correlations serve well for the purposes needed in the industry.

If enhanced oil recovery methods like surfactant and polymer flooding are considered, these concepts come to their limits. There is a need to describe and predict the change in relative permeabilities and residual oil saturation due to changes in surface tension, wetting properties or mobility.

It is clear and known since the early times of reservoir engineering that these quantities are influenced by capillary pressure [1]. In this article the simple relationship between capillary pressure and relative permeability is described. Some examples are given for the illustration of this relationship and how it can be used to predict these properties for cases of low interfacial tension between oil and water.

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2 Single Phase Flow in Porous Media

2.1 Hagen-Poiseuille and the Darcy equation

The flow in porous media is laminar in all flow regimes. The flow rate is directly proportional to the pressure difference applied. This can be expressed by Hagen-Poiseuille's law [2] as it is valid for flow in tubes.

$$q = \frac{\pi \cdot r^4}{8\mu} \cdot \frac{\Delta p}{l} \tag{1}$$

q is the flow rate, *r* the radius of the pipe, *l* the pipe length, μ the viscosity of the flowing medium and Δp is the pressure difference between the two ends of the pipe.

For the flow in porous media the Darcy equation is used to express the relation between flow rate and pressure.

$$q = \frac{k \cdot A}{\mu} \cdot \frac{\Delta p}{l} \tag{2}$$

In the Darcy equation A is the total cross section of the porous medium and the proportionality factor k is called permeability. It is obvious that both equations are equivalent. For pipes with a radius r in Equation (1) the cross sectional area A of such a tube is πr^2 . Equation (1) can then be written as

$$q = \frac{\pi \cdot r^2 \cdot r^2}{8\mu} \cdot \frac{\Delta p}{l} = \frac{r^2}{8} \cdot \frac{A}{\mu} \cdot \frac{\Delta p}{l}$$
(3)

and the permeability factor k becomes

$$k = \frac{r^2}{8} \tag{4}$$

In the Darcy equation the cross sectional area is the total area and not only the effective area of the open pores. Therefore this area has to be multiplied by porosity and we get

$$k = \frac{r^2 \cdot \Phi}{8} \tag{5}$$

for a porous medium, where *r* is the pore radius.

If we consider a porous matrix with a porosity of 30% and pores of $10 \mu m$ diameter or $5 \mu m$ radius having the geometry of tubes it will have a permeability of

$$k = \frac{5 \cdot 5 \cdot 0.3}{8} = 0.9375 \mu m^2 \approx 1D$$

2.2 The concept of the hydraulic radius

Porous media do not have pores of well defined geometry and uniform radii. For the description of the pore geometry, the definition of the hydraulic radius r_H or hydraulic diameter d_H is useful. The hydraulic radius is defined by the ratio of the volume of a pore to its surface as

$$r_{H} = \frac{2 \cdot V_{F}}{S_{V}} \tag{6}$$

where V_F is the volume occupied by the fluid and S_V the inner surface of the porous body. For a pipe e. g. the hydraulic radius would be equal to the radius of the pipe, as shown below.

$$r_{H} = \frac{2 \cdot \pi r^{2} \cdot l}{2\pi r \cdot l} = r$$

KOZENY (1927) and CARMAN (1937, 1938) derived an equation to calculate the permeability of well sorted sands (Equation (7)).

$$k = \frac{1}{k_0 T S_0^2} \cdot \frac{\Phi^3}{(1 - \Phi)^2}$$
(7)

Where k_0 is a shape factor of the sand grains, *T* is the tortuosity factor and S_0 is the specific surface of the porous body. The tortuosity *T* is the relation between the geometric length of a porous medium *l* and the length of the shortest way through the pores l_{Θ} . The tortuosity factor is defined as

$$T = \left(\frac{l_{\Phi}}{l}\right)^2$$

The shape factor for a sand is app. 2, which applies for most porous media being sediments of grains like unconsolidated sands. From the Kozeny-Carman equation the specific surface of a porous medium can be calculated as:

$$S_0 = \sqrt{\frac{\Phi^3}{k_0 T \cdot k \cdot (1 - \Phi)^2}} \tag{8}$$

The specific surface is defined as the ratio of the inner surface S_V to the bulk volume V_B :

$$S_0 = \frac{S_V}{V_B}$$

Using this definition the hydraulic radius can be expressed as:

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Fig. 1 Simplified illustration of capillary pressure, a wetting phase rises in a capillary, whereas the non-wetting phase is depressed

$$r_{H} = \frac{2V_{p}}{S_{V}} = \frac{2\Phi \cdot V}{S_{0}V_{B}} = \frac{2\Phi \cdot V}{S_{0}(1-\Phi)V} = \frac{2\Phi}{S_{0}(1-\Phi)}$$
(9)

V is the total volume. If we insert S_0 from the Kozeny-Carman equation we get

$$r_{H} = \frac{2\Phi}{\sqrt{\frac{\Phi^{3}}{k_{0}T \cdot k \cdot (1-\Phi)^{2}} \cdot (1-\Phi)}}$$
(10)

The hydraulic radius of a porous medium then becomes:

$$r_{H} = \sqrt{\frac{4 \cdot k_{0} T \cdot k}{\phi}} \tag{11}$$

As mentioned above the shape factor for sand grains is about 2 and lies between 2 and 5 for consolidated sands. Combining the shape factor with the tortuosity factor we get 8 for the term " $4k_0T$ " for idealized porous media and about 32 for consolidated sandstones. The concept of the hydraulic radius is e. g. used for the normalization of capillary pressure curves in the Leverett J-function, where the radius in the capillary pressure (Equation (15)) is substituted by the hydraulic radius.

3 Flow of several phases 3.1 Capillary pressure

The wetting properties and the interfacial tension determine the capillary pressure. Capillary pressure is simply the phenomenon of a liquid, which is wetting a solid, being sucked into small capillaries, whereas a non-wetting phase is repelled. This is illustrated in Figure 1.

Oil and water form different, immiscible phases. If we have e. g. an oil droplet in the water phase, this drop will form an ideal sphere. The droplet is kept together by a surface energy. The most stable state of the droplet is that with the minimum surface energy, which is obtained by the form of a globe.

The surface energy E_s is proportional to the surface A

$$E_s = \mathbf{\sigma} \cdot A \tag{12}$$

The constant σ is the specific surface energy or surface tension. The unit is

$$\frac{Energy}{Area} = \frac{Nm}{m^2} = \frac{N}{m}$$

An older unit, still being used, is dynes/cm, which is equal to mN/m.

The surface or interfacial tension of a liquid droplet causes a pressure inside the drop. This pressure can be calculated by Equation (13) derived by LAPLACE and YOUNG [8].

$$p = \frac{2\sigma}{R} \tag{13}$$

p is the pressure in the drop or in general below the curved surface and *R* is the radius of curvature.

If a liquid is in contact with a solid surface it will either wet this surface or not. The strength of the wetting properties of a liquid on a particular solid surface is characterized by the wetting angle. The definition of the wetting or contact angle is shown in Figure 2.

If we "force" a liquid into a small capillary it will form a curved surface in this capillary, as also here the surface energy will minimize. The wettability will spread the liquid a little on the surface. The radius of curvature of a virtual droplet, which we have pressed or which is being sucked into a capillary, is therefore larger than the radius of the capillary. This radius of curvature can be expressed by the wetting angle and the capillary radius as shown in Figure 3.

$$\cos(\Theta) = \frac{r}{R}$$
 or $R = \frac{r}{\cos(\Theta)}$ (14)

If we insert (14) into the Laplace-Young equation (13) we get

$$p_c = \frac{2\sigma \cdot \cos\theta}{r} \tag{15}$$

for the capillary pressure.

3.2 Mobility and relative permeability

If more then one phase are flowing in a porous rock the fraction of flow by each phase should be related to its saturation and its viscosity. Each phase is given a mobility M, which is the ratio of its permeability k_i and its viscosity μ_i .

$$M_i = \frac{k_i}{\mu_i} = \frac{k \cdot k_{ri}}{\mu_i} \tag{16}$$

The phase permeability k_i can be expressed



Fig. 2 Definition of the wetting angle, left: non-wetting, right: wetting fluid

by the absolute permeability of the porous medium and a relative permeability to the phase k_{ri} .

If several phases are flowing, the sum of all flow rates q_i equals the total flow rate q_i and the sum of the fractional flow rates f_i is 1.

$$q_i = \sum q_i \tag{17}$$

The fractional flow of a phase is

$$f_i = \frac{q_i}{\sum q_i} \tag{18}$$

As q_i is proportional to its mobility M

$$f_i = \frac{M_i}{\sum M_i} \tag{19}$$

$$\sum f_i = 1 \tag{20}$$

The flow rate of each phase should be related to its saturation. If no capillary forces or any other interaction between the phases are present the flow rate of a phase can be written as

$$q_{D,i} = \frac{k \cdot A \cdot S_i}{\mu_i} \cdot \frac{p_1 - p_2}{l}$$
(21)

The relative permeability is

$$k_{ri} \equiv S_i \tag{22}$$

If capillary effects are present, which is the case in oil and gas production, the flow of a phase is either supported or reduced by the capillary forces. If a saturation change in a



Fig. 3 Illustration of capillary pressure and wetting angle

porous rock is caused by a flow of fluids the pressure drop caused by capillarity is proportional to the phase saturation and the pressure capillary caused by it.

To illustrate this, we can make a thought experiment as shown in Figure 4 assuming a rock piece having a uniform saturation



Thought experiment. Dividing flow into a "Darcy" component and into a "Capillarity" component

of a wetting and a non-wetting phase. For the Darcy case alone the flow would be according to Equation (21). If we imagine that the saturated core plug is submerged in one of the phases its saturated with, e.g. the non-wetting phase having a pressure p_2 , a flow would start, caused by the capillary forces in the core plug. The pressure inside the core that causes this flow is proportional to the flowing phase saturation times the capillary pressure plus the surrounding pressure p_2 . The pressure analogue to p_1 in Equation (21), that causes the flow is:

$$p_1 = S_i \cdot p_{ci} + p_2 \tag{23}$$

Using this "virtual" pressure in the Darcy equation (21) we get

$$q_{Ci} = \frac{k \cdot A \cdot S_i}{\mu_i} \cdot \frac{S_i \cdot p_{ci} + p_2 - p_2}{l}$$
(24)

$$q_{Ci} = \frac{k \cdot A \cdot S_i}{\mu_i} \cdot S_i \cdot \frac{p_{ci}}{l}$$
(25)

The total flow rate of a phase (the index *i* either denotes the wetting or the non-wetting phase) considering a pressure drop and flow from capillary pressure then is:

$$q_{ri} = q_{Di} \pm q_{Ci} =$$

$$= \frac{k \cdot A \cdot S_i}{\mu_i} \cdot \frac{\Delta p}{l} \pm \frac{k \cdot A \cdot S_i}{\mu_i} \cdot S_i \cdot \frac{p_{ci}}{l} \quad (26)$$

or

$$q_{i} = \frac{k \cdot A}{\mu_{i}} \cdot \frac{\Delta p \cdot \left(S_{i} \pm \frac{p_{ci}}{\Delta p} \cdot S_{i}^{2}\right)}{l}$$
(27)

Where the capillary pressure is a function of the saturation $p_{ci} = p_{ci}(S_i)$.

The flow of the non-wetting phase is increased by capillary forces, whereas the flow of the wetting phase, which is held in the pores by capillary forces, is reduced. The sign before the capillary pressure term should be negative for the wetting and positive for the non-wetting phase.

The relative permeability to a phase can be written as:

$$k_{ri} = \left(S_i \pm \frac{p_{ci}}{\Delta p} \cdot S_i^2\right) \tag{28}$$

Finding an expression for relative permeabilities as in Equation (28) is not surprising, as it is known from empirical derived correlations that relative permeabilities can be expressed by a power law of the phase saturation, in most cases to a power of 2 [3]. It is also not surprising, that relative permeabilities correlate to capillary pressure, but it is remarkable that the relative permeability depends on the pressure gradient. On the other side this is evident as this behaviour of pore filling is used e.g. to measure capillary pressure curves.

Bringing together the pressure drop in the reservoir with capillary pressure is not obvious initially. The pressure drop to be used in these calculations should have a physical meaning. It can e. g. be the pressure difference between two grid blocks in a numerical reservoir simulation model and should be chosen within reasonable bounds.

3.3 Critical or residual saturations

Phase saturations that are bound or trapped in the reservoir due to other forces than capillarity are not considered here. It is observed that some phases need to have a particular saturation before they start flowing. It is obvious that a phase starts flowing at a saturation when the pressure drop is larger than the capillary pressure. This occurs at saturations when $p_{ci}/\Delta p$ becomes smaller or equal to 1 or

$$1 - \frac{p_{ci}}{\Delta p} > 0 \tag{29}$$

The term $(1 - p_{ct}/\Delta p)$ leads to negative saturations in Equation (28), when the pressure



Fia. 5 Oil droplets "trapped" at a water wet pore

drop Δp is smaller than the capillary pressure p_{ci} and thus makes the relative permeability zero or less. So the critical saturation can be calculated from Equation (29) and the equation for the relative permeability becomes

$$k_{ri} = S_i - S_{ic} \pm \left(\frac{p_{ci}}{\Delta p}\right) \cdot (S_i - S_{ic})^2$$
(30)

For a non-wetting phase a meaningful root of Equation (29) occurs when $-p_{ci}/\Delta p$ is equal to 1 at a non zero saturation. This is only the case if capillary pressure becomes negative and means, that the phase is partially wetting. There is no other situation where the relative permeability of the non-wetting phase is zero. This is logical as the non-wetting phase is repelled from the porous medium and therefore is always movable.

Nevertheless a residual saturation for the non-wetting phase is also observed. This can be explained by a mechanism where the non-wetting fluid is trapped at pore throats filled with the wetting phase, so that the capillary pressure of the wetting phase has to be overcome (Fig. 5). This means that the residual saturation of the non-wetting phase is of the same magnitude as that of the wetting phase. It may even be higher as the space before a pore throat may be larger than that of the pore throat itself.

This relation can be described by the tortuosity of the porous medium. Tortuosity T is the relation between the geometric length of a porous medium *l* and the length of the shortest way through the pores l_{Θ} .

$$S_{rN} = T \cdot S_{rW} \tag{31}$$

where N denotes non-wetting and W wetting.

4 Examples

4.1 Pressure regime and typical relative permeabilities in petroleum reservoirs

In Figure 6 typical relative permeabilities as used for the description of the flow of oil and water in sandstone are shown. The data set is the result of a history matching process in numerical reservoir simulation.

For the examples that follow it is essential to have an overview of the pressures and pressure gradients that occur during flow in an oil reservoir and the regimes of the capillary pressures. In Table 1 for a variety of reservoir rocks the average pore radii and capillary pressures are given. In Table 2 the pressure gradients that may occur in a realistic production scenario are calculated. Both the capillary pressures and the pressure gradients lie in the same order of magnitude. In a poor reservoir having small permeabilities the pressure drop in the reservoir during production is higher than in good permeable and porous sand. The same applies for the capillary pressure.

If we look at an oil reservoir having a permeability of 100 mD with a thickness of 10 m



Fig. 6 Typical set of oil/water relative permeabilities and fractional flow curve of water (scaled to 1, zero critical saturations)



and an oil viscosity of 1 mPas and produce this reservoir at a rate of 100 m^3/d , which is a typical rate for such reservoirs, we obtain a pressure gradient of app. 0.06 bar/m at a distance of 50 m from the well.

The average capillary pressure of a water wet sandstone would be about 0.13 bar as shown in Table 1. So the pressure gradient in the reservoir is not high enough to establish a pressure that can overcome the average capillary pressure. Only a part of the pore volume, having larger pores and lower capillary pressure can be drained. More detailed examples are described below.

4.2 Capillary pressure

Capillary pressure curves are usually measured by injecting mercury into the dry sample of the porous medium. A wide range of pressures are usually applied and the mercury saturation is measured by simply weighing the core plug. A typical capillary pressure curve is shown in Figure 7.

The pore size distribution of the sandstone sample can be calculated from the capillary pressure curve by using Equation (15). In Figure 8 the pore size distribution is shown in conventional fractions for the pore radius. These fractions are also given in Table 3. The maximum pore radius lies between 2.5 and 4 μ m and the average pore radius is 1.36 μ m. This pore radius is taken as the hydraulic radius, which is responsible for flow. For the calculation of the permeability using Equation (11) a value of $k_0T \sim 8$ is used.

$$k = \frac{1.36 \cdot 1.36 \cdot 0.19}{32} = 0.0109 \,\mu m^2$$

This is close to the measured value of 10.3 mD.

The mercury capillary pressure curve shown in Figure 7 is used to calculate an oil water capillary pressure curve. This curve is shown in Figure 9. For the calculation of this oil/water capillary curve an interfacial tension of 25 mN/m and a wetting angle of 80° are taken.

The relative permeabilities in the examples below are all calculated using this capillary pressure curve, or derivatives calculated from the pore size distribution with different interfacial tensions or wetting angles. For the calculation of the relative permeabilities





Table 1 Average capillary pressure of porous media

Interfacial tension oil/water Contact angle		25 mN/m 45 degrees				
Permeability mD	Porosity %	Pore radius µm	Capillary pressure bar			
0.1	8	0.100	2.55			
1	10	0.283	0.90			
10	15	0.730	0.35			
100	20	2.000	0.13			
1 000	25	5.657	0.05			
10 000	30	16.330	0.02			

Table 2 Pressure gradient in a reservoir around a well

Flow rate Thickness Viscosity Well radius		100 m ³ /d 10 m 1 mPas 0.1 m	
Distance, m	10 mD	100 mD Pressure, bar/m	1000 mD
0.5	59.305	5.931	0.593
1	25.541	2.554	0.255
2	12.771	1.277	0.128
5	5.627	0.563	0.056
7	3.100	0.310	0.031
10	2.190	0.219	0.022
20	1.277	0.128	0.013
50	0.563	0.056	0.006
100	0.255	0.026	0.003
200	0.128	0.013	0.001

Table 3 Pore radius distribution of sandstone sample

Pore radius interval μm	Frequency %
4	0.37
2.5	30.11
1.6	25.96
1	11.69
0.63	3.65
0.4	5.59
0.25	6.10
0.16	3.28
0.1	2.49
0.063	2.02
0.04	8.75



Fig. 9 Calculated oil/water capillary pressure, interfacial tension 25 mN/m, wetting angle 80 $^\circ$



Fig. 10 Relative permeabilities for a water wet rock, pressure 0.05, 0.1 and 0.2 bar (σ = 25 mN/m, \ominus = 80°)

the capillary pressure is taken from these curves at a particular water saturation. This pressure is inserted into Equation (30). For the saturation in Equation (30) either the water saturation or oil saturation are used and the sign for the capillary pressure is taken as negative for the wetting phase (water) and positive for the non-wetting phase (oil).

4.3 Water wet rock – influence of pressure

For a water wet rock, with porosity and permeability as above (19%, 10 mD) relative permeabilities are calculated using Equation (30). The results are shown in Figure 10.

All calculated relative permeabilities are normalized to 1 for the maximum relative permeability to oil.

The smallest capillary pressure for the above example is 0.008 bar. At a pressure of 0.05 bar a critical water saturation of 0.28 is calculated. The residual oil saturation is almost 0.3 using a tortuosity factor of 1.25. This is reduced by increasing the pressure to about 0.1 bar to a critical water saturation of 0.2 and a residual oil saturation of 0.25. At a pressure of 0.2 bar the irreducible water saturation is 0.12 and the residual oil saturation is 0.18. An increase in pressure drop in a reservoir or core flood is achieved by increasing the flow rate. An increase of oil recovery by applying higher rates has been observed in the laboratory. Also the use of chemicals such as polymers or surfactants for foam in WAG processes tends to increase the pressure drop and the oil recovery.

Regarding this mechanism the increase of oil recovery from EOR methods where the viscosity of the driving medium in increased as in polymer flooding should not only come from an improvement of sweep efficiency. These methods should also be able to reduce the residual oil saturation.

4.4 Wetting angle

The influence of a changing wettability for a water wet rock is shown in Figure 11 for wetting angles of 60 (strongly water wet), 80 (intermediate water wet) and 87 (almost neutral) degrees. A strongly water wet rock shows a higher critical water saturation than a more oil wet rock. A change in wettability may influence the oil recovery significantly. Such processes are micellar flooding or also the so called LoSal process, where the use of fresh water instead of brine for flooding may







Fig. 12 Influence of interfacial tension on relative permeabilities 25 mN/m , 10 mN/m, 0.5 mN/m (Θ = 80°, Δp = 0.2 bar)

change the rock surface and thus the wettability.

Whether such high changes in wettability can be achieved in a reservoir shall not be discussed here. The above example shall only demonstrate the effect of such changes. Change in the wetting angle will change the capillary pressure and thus the relative permeabilities and critical fluid saturations. Methods like alkaline flooding or even simply switching from salt water injection to fresh water may have considerable effects.

4.5 Interfacial tension

The influence of interfacial tension is shown in Figure 12. The decrease of interfacial tension moves the relative permeabilities into the direction of miscible curves and reduces critical saturations.

This behaviour is a typical change in relative permeabilities in surfactant flooding as it is proposed by using the capillary number to calculate increase in oil recovery.

The results from the calculation proposed here show how the increase in oil recovery is obtained by surfactant flooding. In surfactant flooding a chemical, which reduces the interfacial tension between oil and water, is added to the flood water displacing



Fig. 13 Residual saturations as a function of capillary number [9]



Fig. 14 Capillary pressure curve derived from the Hg-capillary curve in Figure 7 for mixed wettability. The fraction of water wet pores is 0.7

the oil. In the ideal case oil and water become one phase and we obtain the idealized relative permeabilities, which are equal to the phase saturation. Though in this case we cannot distinguish any longer between the water and oil phase by definition this picture is used and we can correlate the relative permeabilities to a water and oil component. The characteristics as shown in Figure 12 are the same as used in numerical reservoir simulation, where the relative permeabilities are adjusted in the same way, using measured data and the capillary number (32).

$$N_c = \frac{\mu v_D}{\sigma} \tag{32}$$

In Figure 13 the residual oil and water saturations are shown as a function of the capillary number. These data come from different experimental measurements (LAKE [9]).

4.6 Mixed Wettability

It may occur or even be the general case, that a rock is not purely water or oil wet but of mixed wettability. In Figure 14 a capillary pressure curve, calculated from the above Mercury capillary pressure measurement, by assuming that 70 % of the pores is water wet and 30 % is oil wet, is shown. The contact angle for the water wet portion was chosen 80° the one for the oil wet portion was 140°, which leads to the negative capillary pressures for water as shown in Figure 15. The interfacial tension is 25 mN/m.



Fig. 15 Relative permeabilities for water wet and mixed wettability (σ = 25 mN/m, Δ p = 0.05 bar , Θ _w = 80°, Θ _o = 140°)

In the case of mixed wettability the residual oil saturation is considerably lower as if the rock were water wet as shown in Figure 15. The intersection of the relative permeability curves is shifted to the right side, as observed in laboratory measurements.

5 Calculation procedure

difference are given.

umn E

The relative permeabilities can be calculated

in a spreadsheet. The capillary pressure

curve should be given. In the calculation ex-

ample shown in Appendix A, the capillary

pressure curve shown in Figure 14 for mixed

wettability is used. In the header of the

spreadsheet the parameters used for the cal-

culation of this capillary pressure curve, as

well as interfacial tension and the pressure

First the irreducible or critical water satura-

tion is calculated. This should be the value,

when $1-p/\Delta p$ becomes zero. In the example a saturation value close to this value is taken.

When doing this correctly an interpolated

value should be taken. For the residual oil

saturation the critical water saturation times

the tortuosity factor is used. The critical oil

saturation that can be calculated in this case

with mixed wettability is also marked in col-

In columns F and G the respective saturation

minus the critical saturations are calculated.

These values are used to calculate the rela-

tive permeabilities in columns H and I ac-

In column J the water saturation is copied for

charting in the spreadsheet. The normalized

relative permeabilities are shown in col-

umns K and L. The curves are normalized to

the maximum value calculated for the oil rel-

ative permeability. As we are dealing with

cording to Equation (30).

relative values this normalization is useful and permissible.

A chart of the calculated values is given at the bottom of Appendix A. The relative permeabilities for water above the residual oil saturation are irrelevant and are only shown for completeness. Note that some rows are hidden in the spreadsheet.

6 Conclusions

The Darcy equation as used in reservoir engineering is not an empirical equation as it is often addressed in literature, but can be derived exactly from Hagen-Poiseuilles law. Relative permeabilities to a phase in porous media depend on the saturation of the phase, the capillary pressure and the pressure drop during flow. An equation could be derived to calculate relative permeabilities from capillary pressure curves.

The validity of this equation is shown in different examples. The examples shown comply with observations made in laboratory experiments or observed in the field. It is e. g. possible to describe the effect from change in interfacial tension, wetting angle or flood velocity.

Even though the pressure drop in a reservoir may not be calculated exactly, the method derived here is usable to predict the dependency of relative permeabilities on parameters such as the wetting angle and interfacial tension and is in agreement with observations.

Symbols and units:

r

p

1

k

S

Symbols are used as in the petroleum industry, units are SI units

- flow rate, m³/s q radius, m
- R radius, m
 - pressure, Pa, bar
 - length, m
 - permeability, m²,
 - Darcy $(1 \text{ D} = 0.986923 \cdot 10^{-12} \text{ m}^2)$
- A cross section, m V
 - Volume, m
 - surface, m

S_v	inner surface, m ²
S_0	specific surface, m ⁻¹
$r_{_H}$	hydraulic radius, m
f	fractional flow
Т	tortuosity factor
\mathbf{k}_{0}	shape factor, Kozeny-Carman
μ	viscosity, Pa s
Φ	porosity
Θ	wetting angle
σ	surface tension, N/m
Indices	
i	phase index, wetting or non-wetting
0	oil
W	water
c,C	capillary
D	Darcy
t	total
r	relative
N	non-wetting
W	wetting
Р	pore, porous
F	fluid
В	bulk, for bulk volume of a porous body
φ	related to porosity

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Λ	nondix	Λ.	Evomplo	Coloulation	C	proodchoot
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A	В	C	0	E	F	G	н		J	К	. L
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Mized Vett	ability										
		rad	degree								
Sigma Oil/\	ater		25			k	100				
Theta Oil/R	lock	2.443	140			phi	19				
Theta Vate	r/Rock	1.047	60			delta p	0.05				
Partition V	ater Vet / O	0.7				Swc	0.3148				
						Sor	0.3935				
						Tortuosity fa	1.25				
Sw	So	DCOV	1-nc/dn	1+nc/dn	Sw-Swc	So-Sor	krw	kro	Sw	krw	tro
0.05943	0.94057	1432481307	27 6496261	29 64962615	-0.25536	0.5470825	-2 12356572	9 121894458	0.05943	.0 23279876	
0.06012	0.04007	1226270906	-25 7274161	27 72741612	0.25466	0.5462925	.190797079	9 525420569	0.06012	0.21792496	0.92461074/
0.00013	0.00000	0.512000740	9 27001407	11 27001407	0.23400	0.5463625	0.72422709	2 220002714	0.00013	0.00049064	0.004010144
0.03177	0.30623	0.01000000	-3.27001437	0.000040000	-0.22302	0.5147425	-0.73422703	3.230003714	0.03177	-0.00043064	0.3343703.
0.03333	0.30067	0.430132303	-7.6030461	3.603646036	-0.21346	0.0071020	-0.61407643	2.720365007	0.03333	-0.06740666	0.23022300
0.10668	0.89332	0.370/9/562	-6.41535125	8.41535125	-0.20811	0.4998325	-0.52929316	2.302078349	0.10668	-0.05802448	0.207904575
0.19873	0.80127	0.122854483	-1.45/08966	3.457089655	-0.11606	0.407/825	-0.14915681	0.816363504	0.19873	-0.01635152	0.089494952
0.27293	0.72707	0.057454877	-0.14909755	2.149097548	-0.04186	0.3335825	-0.04387352	0.461450955	0.27293	-0.00480969	0.050587184
0.29113	0.70887	0.053008445	-0.06016891	2.060168907	-0.02366	0.3153825	-0.02425348	0.420833389	0.29113	-0.00265882	0.046134423
0.31479	0.68521	0.048311357	0.033772869	1.966227131	0	0.2917225	0	0.373950378	0.31479	0	0.040994815
0.32214	0.67786	0.047058073	0.058838541	1.941161459	0.00735	0.2843725	0.007299156	0.36048208	0.32214	0.00080018	0.039518335
0.48503	0.51497	0.031942383	0.36115233	1.63884767	0.17024	0.1214825	0.151725136	0.130910613	0.48503	0.016633073	0.014351253
0.52297	0.47703	0.030405749	0.391885023	1.608114977	0.20818	0.0835425	0.181824958	0.087786747	0.52297	0.019932807	0.00962374
0.57057	0.42943	0.02889091	0.42218179	1.57781821	0.25578	0.0359425	0.217977163	0.036688962	0.57057	0.023896041	0.004022077
0.63399	0.36601	0.027277989	0.454440219	1.545559781	0.3192	-0.0274775	0.263613656	-0.0270656	0.63399	0.028899003	-0.002967
0.68756	0.31244	0.010624069	0.78751863	1,21248137	0.37277	-0.0810475	0.343244126	-0.07965177	0.68756	0.037628601	-0.00873193
0 70652	0.29348	0.008238154	0.835236919	1164763081	0.39173	-0.1000075	0.366446711	-0.09835962	0 70652	0.040172216	-0.0107828
0.7337	0.2663	0.005768816	0.884623681	1 115376319	0.41891	.0 1271875	0.398663119	.0 1253211	0.7337	0.043703983	.0.01373849
0.7541	0.2459	0.00244964	0.921007192	1000000007	0.42921	0.1475975	0.425994952	0.14600469	0.7541	0.046700261	0.0160147
0.7070	0.2403	0.001097095	0.031007103	1.000332007	0.45557	0.1020475	0.420334032	0.10000403	0.7391	0.040400201	0.0170974
0.77036	0.22364	0.001037035	0.378038033	1.021341303	0.40007	-0.1630473	0.431016083	-0.16323643	0.77036	0.043443248	-0.01/03/4
0.76366	0.21632	-0.0012054	1.024107313	0.373632067	0.46663	-0.1771675	0.474130313	-0.17732421	0.70360	0.051363754	-0.01350518
0.79652	0.20348	-0.00344115	1.068822354	0.931177046	0.48173	-0.1900075	0.497701316	-0.1924922	0.79652	0.054561179	-0.02110222
0.80195	0.19805	-0.00562685	1.112536911	0.887463089	0.48716	-0.1954375	0.513867807	-0.19973594	0.80195	0.056333452	-0.02189632
0.80939	0.19061	-0.00781254	1.156250869	0.843749131	0.4946	-0.2028775	0.532823519	-0.20930867	0.80939	0.058411498	-0.02294575
0.81515	0.18485	-0.0099315	1.19863005	0.80136995	0.50036	-0.2086375	0.550089045	-0.21728379	0.81515	0.060304255	-0.02382003
0.82052	0.17948	-0.01205046	1.24100923	0.75899077	0.50573	-0.2140075	0.567371203	-0.22504553	0.82052	0.062198834	-0.02467092
0.82532	0.17468	-0.01411937	1.282387328	0.717612672	0.51053	-0.2188075	0.584131682	-0.23232728	0.82532	0.064036225	-0.02546919
0.82952	0.17048	-0.01613822	1.322764342	0.677235658	0.51473	-0.2230075	0.600245435	-0.23905933	0.82952	0.065802717	-0.0262072
0.84017	0.15983	-0.02204461	1.440892137	0.559107863	0.52538	-0.2336575	0.647076875	-0.25772837	0.84017	0.070936676	-0.02825382
0.84332	0.15668	-0.02396335	1.479266985	0.520733015	0.52853	-0.2368075	0.662410338	-0.26368373	0.84332	0.072617628	-0.02890669
0.85346	0.14654	-0.03115446	1.623089243	0.376910757	0.53867	-0.2469475	0.71946892	-0.28494539	0.85346	0.078872752	-0.03123752
0.86126	0.13874	-0.03796183	1,759236532	0.240763468	0.54647	-0.2547475	0.773200396	-0.30401913	0.86126	0.084763138	-0.0333285
0.89306	0,10694	-0.13808681	3,761736233	-1.76173623	0.57827	-0.2865475	1.501784082	-0.5133122	0.89306	0.164635108	-0.05627254
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	A Example C: Mixed Vett Sigma Oil/Theta Oil/F Theta Oil/F Partition V Su 0.05943 0.06013 0.09933 0.05943 0.06013 0.07330 0.27233 0.27133 0.27233 0.27139 0.32214 0.48503 0.75057 0.75057 0.75057 0.75052 0.7541 0.73582 0.80195 0.82522 0.82522 0.82522 0.83246 0.83252 0.83262 0.83262 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.832952 0.8632	A B Etample Calculation Mixed Vettability Sigma Oil/Vater Theta Oil/Rock Theta Viter/Flock Theta Viter/Flock Partition Vater Vet / O So So So 0.05943 0.34057 0.06933 0.34057 0.06933 0.34057 0.06933 0.30067 0.05943 0.34057 0.09933 0.30067 0.09933 0.30067 0.0170 0.08022 0.27233 0.7207 0.2413 0.70867 0.42474 0.65821 0.43274 0.65786 0.45827 0.47703 0.58276 0.21244 0.7087 0.24833 0.7581 0.22458 0.77076 0.22632 0.75852 0.20348 0.80551 0.18965 0.80552 0.17468 0.80554 0.18764 0.81565 0.83274 0.81574 0.45682	A B C Ezample Calculation Mixed Vettability rad Sigma Oil/Vater Theta Dil/Flook 2.443 Theta Vater/Rock 1.047 Partition Vater Vet / O 0.7 Sv So pcov 0.05943 0.34057 1.432481307 0.06013 0.39387 1.336370806 0.09933 0.90667 0.430193205 0.09933 0.9067 0.430193205 0.19873 0.80127 0.258443 0.27233 0.72707 0.05304445 0.3475 0.80127 0.0288493 0.27233 0.72707 0.054348273 0.27234 0.87766 0.044931577 0.2810 0.0272758 0.047058073 0.46503 0.51670 0.028443 0.7057 0.42843 0.02827788 0.86765 0.31244 0.00672858 0.7541 0.2459 0.00372858 0.7552 0.20348 0.000578585 0.7545 0.4485	A B C D Example Calculation	A B C D E Example Calculation	A B C D E F Example Calculation Mized Vettability rad degree	A B C D E F G Example Calculation Intervention Intervention Intervention Intervention Intervention Mixed Vettability Intervention Intervention Intervention Intervention Signa Oil/Vater Intervention Intervention Intervention Intervention Partition Vater Vet / O 0.7 Swe Sor Sor D05943 0.94057 1432461307 -27.6496261 23.64962656 -0.25536 0.8470625 0.05943 0.94057 1432461307 -27.6496261 23.64962656 -0.25646 0.6543322 0.06943 0.94057 1432461307 -27.6496261 23.64962656 -0.25646 0.5453225 0.06943 0.930074 -3.25049471 1.25049479 -0.22040 0.5474567 0.09977 0.930274 0.3270497 -0.20101 0.4998255 0.1600 -0.4998255 0.99770 0.93745477 0.2010 0.9372727 0.9377268 0.937273788 0.948157 </td <td>A B C D E F G H Example Calculation Intel Alexandro Calculation Intel Alexandro</td> <td>A B C D E F G H I Example Calculation Mixed Vettability rad degree .<!--</td--><td>A B C D E F G H I J Example Calculation Mited Vetability rad degree 25 k 1000 1000 1000</td><td>A B C D E F G H I J K Erample Calculation </td></td>	A B C D E F G H Example Calculation Intel Alexandro	A B C D E F G H I Example Calculation Mixed Vettability rad degree . </td <td>A B C D E F G H I J Example Calculation Mited Vetability rad degree 25 k 1000 1000 1000</td> <td>A B C D E F G H I J K Erample Calculation </td>	A B C D E F G H I J Example Calculation Mited Vetability rad degree 25 k 1000 1000 1000	A B C D E F G H I J K Erample Calculation